Math205 Midterm Test

October 23rd, 2007

Answer THREE of the FOUR questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please start a fresh side of paper for each question.

1. (a) Prove, using the contrapositive method, that if $X \cap Y \neq \varnothing$ then $X \cup Y \neq \varnothing$. [5]

   (b) Given these two sets $A := \{3n+1 : n \in \mathbb{Z}\}$ and $B := \{m^2 : m \in \mathbb{Q}\}$ and the universal set $\mathcal{U} := \{1, 2, 4, 5, 6, 7, 9, 10\}$, identify the elements in A and B and plot their Venn diagram. [4]

   (c) Using A and B from part (b) find a set C such that $|C| = 3$ but $A \cap C \neq \varnothing$ and $B \cap C = \varnothing$. Is your choice of C a unique one? [2]

2. (a) Factorise and plot the function $f(x) := x^3 - x$ and hence find the region of the real line where $p(x) :\equiv$ " $x^3 \geq x$ " is true. [2]

   (b) Negate the proposition $q(x) :\equiv$ " $|2x - 1| < 5$ " and hence determine why it is true that [3]

$$\forall x \in \mathbb{Z} \ \ (q(x) \to p(x))$$

   (c) Prove directly that any odd integer subtracted from its cube is a multiple of 4. [6]

3. (a) Build up these set expressions using a series of Venn diagrams: [5]

$$\overline{(A \cup B)} \cap (C \triangle \overline{B}) \ \text{ and } \ \overline{(A \cup B)} \triangle (C \triangle \overline{B})$$

   (b) Use your set algebra formulae to simplify $\overline{(A \cup B)} \cap (C \triangle \overline{B})$. [6]

4. We define a new logic relation using this truth table:

p	q	$p \odot q$
0	0	0
0	1	1
1	0	0
1	1	1

   (a) Using truth tables show that $\odot$ is associative but not commutative. [3]

   (b) Verify using truth tables that $\sim (p \odot q) \equiv ((\sim p) \odot (\sim q))$. [2]

   (c) Simplify $(p \odot q) \to (r \odot q)$ using truth tables. [2]

   (d) Using the fact that algebraically $(p \odot q) :\equiv q$, use algebra on all the expressions in (a) to (c). [4]