Cape Breton University MATH205

Discrete Mathematics

11th December 2007 Time : 3 hours

Clearly write your answers to the questions showing all working and checks and indicate what each mathematical calculation is doing. The best FIVE answers will be counted.

Q1. (a) Prove by induction that [7]

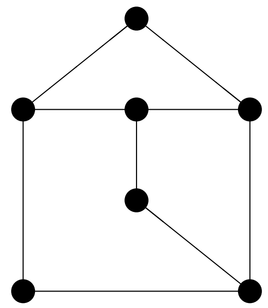
$$\sum_{i=1}^{n}(i+13)(i-11) = \frac{n(n+23)(2n-37)}{6}$$

(b) Use the above formula to find $\displaystyle\sum_{i=p+1}^{2p+1}(i+13)(i-11)$ for any $p \geq 1$. Check if your answer works for $p = 1, 0$ and -1. [5]

Q2. (a) Prove by contradiction that the absolute value of $3x - 25$ is at least 2 for all odd numbers x. [6]

(b) If $p(x) :\equiv$ "$2x^2 \geq 7x + 15$" and $q(x) :\equiv$ "$4(x+1) < x$", plot both on the number line and decide whether these statements are true or false:
$\forall x \in \mathbb{R}, (q(x) \to p(x))$, $\exists x \in \mathbb{Z}; (((\sim p(x)) \wedge q(x))$. [6]

Q3. (a) Given these three lists of the edges in some graphs, draw them carefully without edges crossing and then prove that they are non-isomorphic but one is the same as G, and give the isomorphism to G. [8]

$G :=$ 

$E(G_1) := \{ad, ae, af, bd, be, bg, cd, cf, cg\}$

$E(G_2) := \{hi, hj, il, in, jk, jn, kl, lm, mn\}$

$E(G_3) := \{pq, pv, qr, qs, rs, rt, st, tu, uv\}$

(b) Find a graph which has the same valency sequence as those in part (a) but which is 1-connected or disconnected. [2]

(c) What is the minimum number of colours required for each of the three graphs? [2]

Q4. On a shelf there are 6 red books, 5 green books, 2 blue books and 3 white books and those of the same colour are identical.

(a) How many different stacks of five books are there which are the same if you turn the stack upside-down? Count and list all stacks with a blue book on top. [5]

(b) Four people each pre-select a favourite book and then try to buy them as a group. Count the ways that this will not be possible, and hence count the number of ways it can happen. [4]

(c) If five people split the entire shelf between them, how many must at least one of the people have? Must any of them have two copies of some book? [3]

Q5. (a) Given these three relations between the sets $\{a, b, c\}$ and $\{v, w, x, y, z\}$ give the reason why two are not functions and why the other is. [3]

$$\begin{aligned} Q &:= \{(x, b), (z, c), (v, a)\} \\ R &:= \{(v, x), (x, z), (w, w), (z, y), (y, v)\} \\ S &:= \{(a, w), (a, z), (b, w), (b, x), (c, v)\} \end{aligned}$$

(b) Evaluate the compositions $Q \circ S$ and $Q \circ R$. Display them in table form and hence determine whether either are functions and whether they are 1-1 or onto. [6]

(c) Prove that if T is everywhere defined then $(T^{-1} \circ T)$ has the reflexive property. [3]

Q6. (a) Simplify this expression using the logic rules: $(p \to q) \leftrightarrow p$. [7]

(b) Create truth tables for $q \wedge (p \to r)$, $q \wedge p$ and $q \wedge r$ and hence establish that distributivity does not hold between these two symbols. [5]

Q7. (a) Use the contrapostive method to prove that if $A \subseteq B$ then $|B| \geq |A|$. [5]

(b) Explain why, even if $A \cap B \neq \varnothing$, if $|B| > |A|$ then we do not have to have $A \subset B$. [2]

(c) Identify the areas which are $(D \cap \overline{E}) \cup (F \cap \overline{D})$ and $\overline{(E \cup (\overline{D} \cap \overline{F}))}$ by shading Venn diagrams. When do these two sets contain exactly the same elements? [5]

END OF QUESTION PAPER