Cape Breton University

MATH205

Discrete Mathematics

31st October 2006 Time : 1.5 hours

Clearly write your answers to the questions showing all working and checks and indicate what each mathematical calculation is doing. The best THREE answers will be counted.

Q1. (a) Prove, using Venn diagrams, that [5]

$$(((A \cap \overline{C}) \cap B) \cup C) \subseteq (A \cup C)$$

(b) Given these sets, and the universal set of the first 15 positive numbers, insert the elements into their Venn diagram [2]

$$\begin{aligned} A &:= \{2, 3, 5, 7, 11, 13\} \ , \\ B &:= \{\text{numbers one less than a power of two}\} \ , \\ C &:= \{\text{numbers less than 7}\} \end{aligned}$$

(c) Verify the relation in (a) and also find what these are: $|A \cap (B \cup C)|, \overline{(B \cup \overline{C})}$ [4]

Q2. (a) Prove, by the direct method and then by the contradiction method, that two odd numbers add to make an even one. [8]

(b) Disprove these statements by finding different counterexamples for each: "Two sets with the same cardinality are equal", "A universal set with two elements in has two subsets" "The intersection of any set with itself is never empty" [3]

Q3. Prove by induction that $\sum_{i=-1}^{n} (9i - 4)(i + 1) = (n + 1)(n + 2)(3n - 2)$ [11]

Q4. (a) Simplify this logic expression: $((p \wedge q) \rightarrow r) \rightarrow (\sim q)$ [6]

(b) Using part (a) and given $p(x) :\equiv$ " $|2x + 3| > 1$ ", $q(x) :\equiv$ " $x \leq -2$ " $r(x) :\equiv$ " $x^2 < x$ ", plot these regions on a real line and establish whether it is true that: [5]

$$\forall x \in \mathbb{R}; (((p(x) \wedge q(x)) \rightarrow r(x)) \rightarrow (\sim q(x)))$$

END OF QUESTION PAPER