

# Math205 Handout 1: Sets and Logic Formulae 2006

- Basic Definitions

Set Theory	Notation	Definition	Logic	Notation
Complement	$\bar{A}$	not in	Not	$\sim p$
Intersection	$A \cap B$	in both	And	$p \wedge q$
Union	$A \cup B$	in either <i>or</i> both	Or	$p \vee q$
Disjoint Union	$A \Delta B$	in either <i>but not</i> both	Xor	$p \underline{\vee} q$
		If $p$ is true then $q$ is true	Implies	$p \rightarrow q$
		If $p$ and $q$ are the same	If and only if	$p \leftrightarrow q$
Universal Set	$\mathcal{U}$	Always true	Tautology	$T_0$
Empty Set	$\emptyset$	Never true		$\sim (T_0)$

Truth table:

$p$	$q$	$\sim p$	$p \wedge q$	$p \vee q$	$p \underline{\vee} q$	$p \rightarrow q$	$p \leftrightarrow q$	$T_0$
0	0	1	0	0	0	1	1	1
0	1	1	0	1	1	1	0	1
1	0	0	0	1	1	0	0	1
1	1	0	1	1	0	1	1	1

- Basic Relationships

Double Negation	$\sim(\sim p) \equiv p$	
De Morgan	$\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$	$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$
Commutative	$(p \vee q) \equiv (q \vee p)$	$(p \wedge q) \equiv (q \wedge p)$
Associative	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Idempotent	$p \vee p \equiv p$	$p \wedge p \equiv p$
Absorbtion	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Identity	$p \vee (\sim T_0) \equiv p$	$p \wedge T_0 \equiv p$
Cancellation	$p \vee (T_0) \equiv T_0$	$p \wedge (\sim T_0) \equiv (\sim T_0)$
Inverse	$p \vee (\sim p) \equiv T_0$	$p \wedge (\sim p) \equiv (\sim T_0)$
Xor	$p \underline{\vee} q \equiv (\sim(p \wedge q)) \wedge (p \vee q)$ $\equiv ((\sim p) \wedge q) \vee (p \wedge (\sim q))$	
Implication	$p \rightarrow q \equiv (\sim p) \vee q$	
Biconditional	$p \leftrightarrow q \equiv ((\sim p) \vee q) \wedge (p \vee (\sim q))$ $\equiv (p \rightarrow q) \wedge (q \rightarrow p)$ $\equiv (\sim(p \vee q)) \vee (p \wedge q)$	

- Quantifiers

There Exists	$\exists$
For All	$\forall$
Negation	$\sim(\forall x (p(x))) \equiv \exists x (\sim p(x))$ $\sim(\exists x (p(x))) \equiv \forall x (\sim p(x))$
Contrapositive	$\forall x (p(x) \rightarrow q(x)) \equiv \forall x ((\sim q(x)) \rightarrow (\sim p(x)))$
Equivalences	$\forall x (p(x) \wedge q(x)) \equiv (\forall x p(x)) \wedge (\forall x q(x))$ $\exists x (p(x) \vee q(x)) \equiv (\exists x p(x)) \vee (\exists x q(x))$
Implications	$\exists x (p(x) \wedge q(x)) \rightarrow (\exists x p(x)) \wedge (\exists x q(x))$ $(\forall x p(x)) \vee (\forall x q(x)) \rightarrow \forall x (p(x) \vee q(x))$