

# Math105 2005 Test 3

November 2nd, 2005

Answer all questions and give complete reasons and checks for your answers. On your answer sheet please write and sign a declaration that what you are handing in is your own work. Anyone who plagiarises or lets someone else copy their work will receive zero and be reported to their Dean.

1. Using the contrapositive method and showing all steps, prove that if  $A \cup B \subseteq A$  then  $B \subseteq A$ .
2. Using induction prove that

$$\sum_{i=1}^n (i-3)(i+2) = \frac{n(n^2-19)}{3}$$

3. (a) Solve this recurrence relation  $a_{n+1} = 3a_n + 10a_{n-1}$  if  $a_0 = 7$  and  $a_1 = 14$ .  
(b) Why is your final answer for  $a_n$  always a multiple of 7?
4. Given 250 students in 23 classrooms at one time, explain why at least one classroom must have at least 11 students in it. If there were just 13 classrooms and the same number of students, what can you say about the numbers of students in any particular classroom?