

DISCRETE MATHEMATICS

20th October 2005

Student Name:

Time : 1.25 hours

Registration Number:

Clearly write your answers to the questions showing all working and checks and indicate what each mathematical calculation is doing. The best THREE questions will be counted.

Q1. Prove, giving all steps and using the contradiction method, that (for any two real numbers a and b) if $a \times b = 0$ then either a or b must be zero. [11]

- Q2.** (a) Expand $\sim (p \wedge (q \wedge ((\sim p) \vee r)))$ to an expression with just one of each letter. [7]
- (b) Calculate the truth tables of both the above expression and your final expression, showing the links between your columns. [4]

- Q3.** (a) Translate this sentence into mathematical symbols: “For all integers x either $p(x)$ is true or $q(x)$ is false.” [2]
- (b) For $p(x) :\equiv “x^2 \leq 11”$ and $q(x) :\equiv “2x - 6 < 0”$ evaluate $p(1)$, $q(4)$ and “ $p(-4) \vee q(-4)$ ”. [3]
- (c) Negate the sentence in part (a) by stating and using appropriate rules of logic. [4]
- (d) Find an x for which the sentence in part (c) is true and check that this value makes (a) not true. [2]

Q4. Three sets are defined as follows with respect to this universal set:

$$\mathcal{U} := \{\text{dog, moose, beaver, cat, lion, penguin, aardvark, hippo, kangaroo}\}$$

- (a) If C is the set of words which include the letter o, B is the set of words with at most four letters in and A is the set of words from the first third of the universal set, list the contents of the three sets and draw a Venn diagram of their relationship. [4]
- (b) By sketching the individual Venn diagrams identify the contents of the two sets [4]

$$\overline{(A \cap B) \cup C} \quad \text{and} \quad (A \Delta B) \cap C$$

- (c) Evaluate the cardinalities of the sets in the 3 set inclusion-exclusion formula and check that the formula holds for these particular sets. [3]

END OF QUESTION PAPER