Cape Breton University

Math105

DISCRETE MATHEMATICS

December 2005

Time : 3 hours

Your final mark will be the total of your best FIVE questions. Give all working and reasoning for each question, do not erase mistakes but instead cross them out and continue, and start a fresh side of paper for each question.

Q1. (a) Using Venn diagrams show that [5]

$$(A \triangle C) \cup B = ((A \cup B) \triangle (\overline{B} \cap C))$$

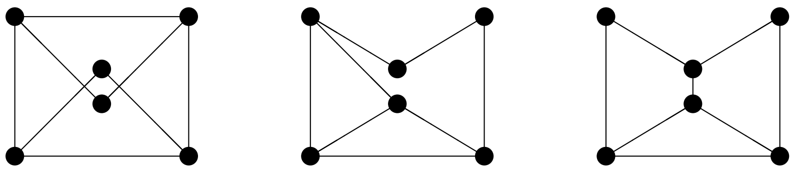
(b) If the universal set is the digits from 0 to 9 and draw the Venn diagram of these three sets and determine the elements in these sets: $A \cap B \cap C$, $\overline{(A \cup C)} \cap B$, $(B \cup C) \cap A$ and $(\overline{B} \cap A) \cup (\overline{C} \cap C)$. [7]

$$\begin{aligned} A &:= \{\text{odd numbers}\} \\ B &:= \{\text{numbers one less than a square of an integer}\} \\ C &:= \{\text{numbers which are not integer multiples of three}\} \end{aligned}$$

Q2. (a) Prove by contradiction that if x is an even number then $2 - 3x + x^2$ is not negative. [10]

(b) Give an example of an x which is not even which has $2 - 3x + x^2$ positive and an x which makes it negative. [2]

Q3. (a) Given these graphs find their symmetries and hence their diameters. Find a Hamiltonian Cycle in each one and show them all non-isomorphic. [8]



(b) Draw this graph and prove it is not isomorphic to any of those above. [4]

$$E(G) := \{ab, ef, dc, da, ae, ec, cb, fb\}$$

Q4. (a) Given $5a_n = 2a_{n+1} - 3a_{n-1}$, $a_1 = -42$ and $a_0 = 98$, find a_2 and a_3. [2]

(b) Solve the above equation for a general a_n. [7]

(c) Prove by induction that $a_{n+1} > a_n$ if $n \geq 3$. [3]

Q5. In a store there is a display of winter clothing which has in it four scarves (1 red, 2 blue and 1 green), 7 pairs of gloves (4 red and 3 blue) 6 hats (2 red, 1 blue and 3 green) and 5 coats (3 blue and 2 green). We count the gloves as one item and the items of the same colour as indistinguishable.

(a) How many ways are there to choose one of each type of clothing? [2]

(b) How many ways are there to choose two items of blue clothing? [3]

(c) How many ways are there to choose four items of all different colours? [1]

(d) How many ways are there to choose the first and second items to be bought? [2]

(e) How many ways are there to choose a group of 6 items, irrespective of colour? [4]

Q6. Use induction to prove that [12]

$$\sum_{i=2}^{n}((3i+1)\times(i-2)) = (n-2)(n-1)(n+2)$$

Q7. (a) Simplify the logic expression $p \to ((r \to (\sim q)) \wedge p)$ to an expression with one $\sim$ symbol and one p, q and r. Check your answer with a truth table. [7]

(b) Prove that $(((\sim q) \wedge p) \wedge ((\sim p) \vee q)) \equiv (\sim T_0)$ using logic rules. [5]

Q8. (a) Draw or make a table of this relation and explain why it satisfies all properties of a partially ordered set [4]

$$R := \{(x,w),(x,z),(y,x),(y,z),(y,w),(v,w),(y,v),(y,u)\}$$

(b) Why can no partially ordered set be onto? [2]

(c) Explain which of the properties one-to-one, everywhere defined and uniquely defined S is if it is a relation from $\{a,b,c,d,e,f\}$ to $\{4,5,6,7,8\}$. [3]

$$S := \{(a,7),(b,4),(e,6),(c,4),(f,8)\}$$

(d) Explain why that if S is a relation from a set X to a set Y that if S contains fewer relations than the number of elements in either set then it cannot be everywhere defined or onto. Give a relation which has more relations than the total number of elements in both X and Y which still isn't everywhere defined or onto. [4]

END OF QUESTION PAPER