$\underline{n}$ is normal to direction so
$$\left(\begin{array}{ccc|c} 3 & 3 & 7 & 0 \\ 4 & 5 & 5 & 0 \end{array}\right)$$
$R2 \Leftarrow R2 - \frac{4}{3}R1$
$$\left(\begin{array}{ccc|c} 3 & 3 & 7 & 0 \\ 0 & 1 & -\frac{13}{3} & 0 \end{array}\right)$$
$R1 \Leftarrow \frac{R1}{3} - R2$
$$\left(\begin{array}{ccc|c} 1 & 0 & \frac{20}{3} & 0 \\ 0 & 1 & -\frac{13}{3} & 0 \end{array}\right)$$

so $n = \begin{pmatrix} -20 \\ 13 \\ 3 \end{pmatrix}$

check $-60+39+21=0$
$-80+65+15=0$

$\underline{n} \cdot \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = -40+39-3 = -4$ so P has eqn $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -20 \\ 13 \\ 3 \end{pmatrix} = -4$

(b) $\begin{pmatrix} 6+7t \\ -1+11t \\ 9+t \end{pmatrix} \cdot \begin{pmatrix} -20 \\ 13 \\ 3 \end{pmatrix} = -4$ so $-120-140t-13+143t+27+3t=-4$

$6t = -4+106 = 102$

so $t = 17$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 125 \\ 186 \\ 26 \end{pmatrix}$

check $\begin{pmatrix} 125 \\ 186 \\ 26 \end{pmatrix} \cdot \begin{pmatrix} -20 \\ 13 \\ 3 \end{pmatrix} = -2500+2418+78 = \underline{-4}$ ✓

(c) $\begin{pmatrix} 3 & 4 \\ 3 & 5 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} s \\ k \end{pmatrix} = \begin{pmatrix} 125 \\ 186 \\ 26 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 123 \\ 183 \\ 27 \end{pmatrix}$ so $\left(\begin{array}{cc|c} 3 & 4 & 123 \\ 3 & 5 & 183 \\ 7 & 5 & 27 \end{array}\right)$ $R_2 \Leftarrow R_2 - R_1$, $R_3 \Leftarrow R_3 - \frac{7}{3}R_1$

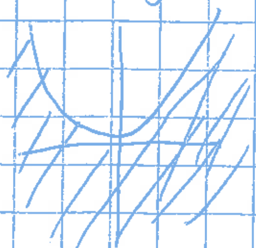
$$\left(\begin{array}{cc|c} 3 & 4 & 123 \\ 0 & 1 & 60 \\ 0 & -\frac{13}{3} & -260 \end{array}\right)$$
$R1 \Leftarrow R1 - 4R2$
$R3 \Leftarrow R3 + \frac{13}{3}R2$

$$\left(\begin{array}{cc|c} 3 & 0 & -117 \\ 0 & 1 & 60 \\ 0 & 0 & 0 \end{array}\right)$$
$R_1 \Leftarrow R_1 \times \frac{1}{3}$

$$\left(\begin{array}{cc|c} 1 & 0 & -39 \\ 0 & 1 & 60 \\ 0 & 0 & 0 \end{array}\right)$$
$s = -39$
$k = 60$

check
$$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + -39 \times \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} + 60 \times \begin{pmatrix} 4 \\ 5 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} 2 - 117 + 240 \\ 3 - 117 + 300 \\ -1 - 273 + 300 \end{pmatrix} = \begin{pmatrix} 125 \\ 186 \\ 26 \end{pmatrix}$$ ✓

Q2 $y \leq x^2$



A4 true $x=0$ $y=0$ $x^2=0 \geq 0 = y$

A1 false $x_1=2$ $y_1=3$ $3 \leq 2^2$

$x_2=-1$ $y_2=0$ $0<(-1)^2=1$

But $x_1+x_2=1$ $y_1+y_2=3 > 1^2$

so $y \not\leq x^2$

M1 false $x=2$ $y=3$ $\alpha=-1$ $\alpha\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-2\\-3\end{pmatrix}$ $-3 \leq (-2)^2$ works in S

$x=0$ $y=-5$ $\alpha=-1$ $\alpha\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\5\end{pmatrix}$ $5>0^2$ outside S

Q3 a) i)
$$\left(\begin{array}{ccc|c} 2 & 1 & 3 & 0\\ 1 & 2 & -4 & 0\\ 0 & 3 & 1 & 0\\ -2 & 2 & 2 & 0\end{array}\right) \begin{array}{l} R_1 \Leftarrow R_1-2R_2\\ R_4 \Leftarrow R_4+2R_2\end{array} \left(\begin{array}{ccc|c} 0 & -3 & 11 & 0\\ 1 & 2 & 4 & 0\\ 0 & 3 & 1 & 0\\ 0 & 6 & -6 & 0\end{array}\right) \begin{array}{l} R_1 \Leftarrow R_1-11R_3\\ R_2 \Leftarrow R_2-4R_3\\ R_4 \Leftarrow R_4+6R_3\end{array} \rightarrow$$

$$\rightarrow \left(\begin{array}{ccc|c} 0 & -36 & 0 & 0\\ 1 & -10 & 0 & 0\\ 0 & 3 & 1 & 0\\ 0 & 24 & 0 & 0\end{array}\right)$$ we can pivot a third time so $a=b=c=0$

ii)
$$\left(\begin{array}{cccc|c} 2 & 1 & 0 & -2 & 0\\ 1 & 2 & 3 & 2 & 0\\ 3 & -4 & 1 & 2 & 0\end{array}\right) R_2 \Leftarrow R_2-3R_3 \left(\begin{array}{cccc|c} 2 & 1 & 0 & -2 & 0\\ -8 & 14 & 0 & -4 & 0\\ 3 & -4 & 1 & 2 & 0\end{array}\right) \begin{array}{l} R_2 \Leftarrow R_2-14R_1\\ \longrightarrow R_3 \Leftarrow R_3+4R_1\end{array}$$

$$\rightarrow \left(\begin{array}{cccc|c} 2 & 1 & 0 & -2 & 0\\ -36 & 0 & 0 & 24 & 0\\ 11 & 0 & 1 & -6 & 0\end{array}\right) R_2 \Leftarrow R_2 \times \frac{1}{24} \left(\begin{array}{cccc|c} 2 & 1 & 0 & -2 & 0\\ -3/2 & 0 & 0 & 1 & 0\\ 11 & 0 & 1 & -6 & 0\end{array}\right) \begin{array}{l} R_1 \Leftarrow R_1+2R_2\\ R_3 \Leftarrow R_3+6R_2\end{array} \left(\begin{array}{cccc|c} -1 & 1 & 0 & 0 & 0\\ -3/2 & 0 & 0 & 1 & 0\\ 2 & 0 & 1 & 0 & 0\end{array}\right)$$

Thus $\underline{n}=\begin{pmatrix}2\\2\\-4\\3\end{pmatrix}$ and $\begin{pmatrix}w\\x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}2\\2\\-4\\3\end{pmatrix}=0$

b) $\underline{v}_1 \cdot \underline{v}_2 = 2+2+0-4=0$ so already orthog $\underline{e}_1=\underline{v}_1$, $\underline{e}_2=\underline{v}_2$

$$\underline{e}_3 = \underline{v}_3 - \frac{\underline{e}_1\cdot\underline{v}_3}{\underline{e}_1\cdot\underline{e}_1}\underline{e}_1 - \frac{\underline{e}_2\cdot\underline{v}_3}{\underline{e}_2\cdot\underline{e}_2}\underline{e}_2 = \begin{pmatrix}3\\-4\\1\\2\end{pmatrix} - \frac{-2}{9}\begin{pmatrix}2\\1\\0\\2\end{pmatrix} \cancel{+} \frac{2}{18}\begin{pmatrix}1\\2\\3\\2\end{pmatrix} = \frac{1}{18}\begin{pmatrix}54+8-2\\-72+4-4\\18+0-6\\36-8-4\end{pmatrix} = \frac{12}{18}\begin{pmatrix}5\\-6\\1\\2\end{pmatrix}$$