

test 1 2018

$$\text{Q11} \quad \text{Row 3} \leftarrow \text{Row 3} \times \frac{1}{3}$$

$$\begin{pmatrix} 5 & 8 & 5 & | & 2 \\ 4 & 7 & 3 & | & 5 \\ \textcircled{1} & 1 & 2 & | & -3 \end{pmatrix}$$

$$\text{Row 1} \leftarrow \text{Row 1} - 5 \text{Row 3}$$

$$\text{Row 2} \leftarrow \text{Row 2} - 4 \text{Row 3}$$

$$\begin{pmatrix} 0 & 3 & -5 & | & 17 \\ 0 & 3 & -5 & | & 17 \\ 1 & 1 & 2 & | & -3 \end{pmatrix}$$

$$\text{Row 1} \leftarrow \text{Row 1} - \text{Row 2}$$

$$\begin{pmatrix} 0 & 0 & 0 & | & 0 \\ 0 & 3 & -5 & | & 17 \\ 1 & 1 & 2 & | & -3 \end{pmatrix}$$

$$\text{Row 2} \leftarrow \text{Row 2} \times \frac{1}{3}$$

$$\begin{pmatrix} 0 & 0 & 0 & | & 0 \\ 0 & 1 & -\frac{5}{3} & | & \frac{17}{3} \\ 1 & 1 & 2 & | & -3 \end{pmatrix}$$

$$\text{Row 3} \leftarrow \text{Row 3} - \text{Row 2}$$

$$\begin{pmatrix} 0 & 0 & 0 & | & 0 \\ 0 & 1 & -\frac{5}{3} & | & \frac{17}{3} \\ 1 & 0 & \frac{11}{3} & | & -\frac{26}{3} \end{pmatrix}$$

So since column 3 was not pivoted

$$z := k$$

$$\text{Row 2: } y = \frac{5}{3}k + \frac{17}{3}$$

$$\text{Row 3: } x = -\frac{11}{3}k - \frac{26}{3}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{11}{3} \\ \frac{5}{3} \\ 1 \end{pmatrix} k + \begin{pmatrix} -\frac{26}{3} \\ \frac{17}{3} \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -11 \\ 5 \\ 3 \end{pmatrix} \frac{k}{3} + \begin{pmatrix} -26 \\ 17 \\ 0 \end{pmatrix} \frac{1}{3}$$

$$\text{check } \begin{pmatrix} 5 & 8 & 5 \\ 4 & 7 & 3 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} -11 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -55 + 40 + 15 \\ -44 + 35 + 9 \\ -11 + 5 + 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \checkmark$$

$$\frac{1}{3} \begin{pmatrix} 5 & 8 & 5 \\ 4 & 7 & 3 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} -26 \\ 17 \\ 0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -130 + 136 \\ -104 + 119 \\ -26 + 17 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 6 \\ +15 \\ -9 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} \checkmark$$

$$(b) \quad -\frac{11}{3}k - \frac{26}{3} \quad \text{Try } k = 1 \quad x = \frac{39}{3} \quad \text{---}$$

$$y = \frac{-5 + 17}{3} = \frac{12}{3} = 4$$

$$\text{Try } k = -1$$

$$x = \frac{11 - 26}{3} = \frac{-15}{3} = -5$$

$$y = \frac{-5 + 17}{3} = \frac{12}{3} = 4$$

$$z = -1$$

all integers

Test 1 2018

Q2

$$\begin{array}{ccc|c} \text{"x"} & \text{"y"} & \text{"z"} & \\ \hline -4 & -3 & 4 & 5 \\ 4 & -1 & -3 & 8 \\ \textcircled{1} & 2 & -1 & -5 \end{array}$$

$$\text{Row 1} \leftarrow \text{Row 1} + 4\text{Row 3}$$

$$\text{Row 2} \leftarrow \text{Row 2} - 4\text{Row 3}$$

$$4\text{Row 3} = (4 \ 8 \ -4 \ : -20)$$

$$\begin{array}{ccc|c} 0 & 5 & 0 & -15 \\ 0 & -9 & 1 & 28 \\ 1 & 2 & -1 & -5 \end{array}$$

$$\text{Row 1} \leftarrow \text{Row 1} \times \frac{1}{5}$$

$$\begin{array}{ccc|c} 0 & \textcircled{1} & 0 & -3 \\ 0 & -9 & 1 & 28 \\ 1 & 2 & -1 & -5 \end{array}$$

$$\text{Row 2} \leftarrow \text{Row 2} + 9\text{Row 1}$$

$$\text{Row 3} \leftarrow \text{Row 3} - 2\text{Row 1}$$

$$\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 0 & 0 & \textcircled{1} & 1 \\ 1 & 0 & -1 & 1 \end{array}$$

$$\text{Row 3} \leftarrow \text{Row 3} + \text{Row 2}$$

$$\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 2 \end{array}$$

$$\text{Row 1} : y = -3$$

$$\text{Row 2} : z = 1$$

$$\text{Row 3} : x = 2$$

Check $x=2 \ y=-3 \ z=1$

$$-8 + 9 + 4 = 5 \quad \checkmark$$

$$8 + 3 - 3 = 8 \quad \checkmark$$

$$2 - 6 - 1 = -5 \quad \checkmark$$

Most efficient as first pivot gives 3 zeros, no zeros are destroyed