Q1

(a) $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \\ 1 & 5 \\ 1 & 8 \end{pmatrix}$ $\quad B = \begin{pmatrix} 13 \\ 10 \\ 11 \\ 5 \\ 6 \end{pmatrix}$ $\quad A^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 5 & 8 \end{pmatrix}$

$A^TA = \begin{pmatrix} 1+1+1+1+1 & 1+2+4+5+8 \\ & 1+4+16+25+64 \end{pmatrix} = \begin{pmatrix} 5 & 20 \\ 20 & 110 \end{pmatrix}$ $\quad A^TB = \begin{pmatrix} 13+10+11+5+6 \\ 13+20+44+25+48 \end{pmatrix} = \begin{pmatrix} 45 \\ 150 \end{pmatrix}$

so [illegible] is solution to $\left(\begin{array}{cc|c} 5 & 20 & 45 \\ 20 & 110 & 150 \end{array}\right)$

$R1 \Leftarrow R1 \div 5 \quad R2 \Leftarrow R2 \div 10$

$\left(\begin{array}{cc|c} 1 & 4 & 9 \\ 2 & 11 & 15 \end{array}\right)$

$R2 \Leftarrow R2 - 2R1$

$\left(\begin{array}{cc|c} 1 & 4 & 9 \\ 0 & 3 & -3 \end{array}\right)$

$R2 \Leftarrow R2 \times \frac{1}{3}$

$\left(\begin{array}{cc|c} 1 & 4 & 9 \\ 0 & 1 & -1 \end{array}\right)$

$R1 \Leftarrow R1 - 4R2$

$\left(\begin{array}{cc|c} 1 & 0 & 13 \\ 0 & 1 & -1 \end{array}\right)$

or $\frac{1}{550-400}\begin{pmatrix} 110 & -20 \\ -20 & 5 \end{pmatrix}\begin{pmatrix} 45 \\ 150 \end{pmatrix}$

$= \frac{1}{150}\begin{pmatrix} 110 & -20 \\ -20 & 5 \end{pmatrix}\begin{pmatrix} 45 \\ 150 \end{pmatrix}$

$= \frac{1}{150}\begin{pmatrix} 4950-3000 \\ -900+750 \end{pmatrix} = \begin{pmatrix} 13 \\ -1 \end{pmatrix}$

$13 \times 150 = 1950$

so

$f(x) = 13 - x$

(b)

x_j	1	2	4	5	8
y_j	13	10	11	5	6
$f(x_j)$	12	11	9	8	5
$f(x_j)-y_j$	-1	1	-2	3	-1

$\Sigma = -1+1-2+3-1 = 0$

(c) $f(x) = 13 - x < 3 \quad x > 13-3 = \underline{\underline{10}}$

Q2 (a) $2^3 - 2^2 - 14\times 2 + 24 = 8-4-28+24 = 4-4 = 0$

$$
\begin{array}{r|l}
 & x^2 + x - 12 \\
\hline
x-2 & x^3 - x^2 - 14x + 24 \\
 & \underline{x^3 - 2x^2} \\
 & \quad x^2 - 14x \\
 & \quad \underline{x^2 - 2x} \\
 & \qquad -12x + 24 \\
 & \qquad \underline{-12x + 24} \\
 & \qquad\qquad 0
\end{array}
$$

so $(x^2+x-12) = (x+4)(x-3)$

so other 2 roots are -4 and $+3$

(b) $\lambda_1 = 2 \quad \lambda_2 = -4 \quad \lambda_3 = 3$

$\underline{v}_1 = \begin{pmatrix}4\\2\\1\end{pmatrix} \quad \underline{v}_2 = \begin{pmatrix}16\\-4\\1\end{pmatrix} \quad \underline{v}_3 = \begin{pmatrix}9\\3\\1\end{pmatrix}$

so
$$\begin{pmatrix}b_{k+1}\\b_k\\b_{k-1}\end{pmatrix} = \underbrace{\begin{pmatrix}4&16&9\\2&-4&3\\1&1&1\end{pmatrix}\begin{pmatrix}2^{k-1}&0&0\\0&(-4)^{k-1}&0\\0&0&3^{k-1}\end{pmatrix}\begin{pmatrix}4&16&9\\2&-4&3\\1&1&1\end{pmatrix}^{-1}}_{=\begin{pmatrix}1&14&-24\\1&0&0\\0&1&0\end{pmatrix}^{k-1}}\begin{pmatrix}185\\73\\51\end{pmatrix}$$

$$= \begin{pmatrix}2^{k+1}&(-4)^{k+1}&3^{k+1}\\2^k&(-4)^k&3^k\\2^{k-1}&(-4)^{k-1}&3^{k-1}\end{pmatrix}\begin{pmatrix}59\\3\\-11\end{pmatrix}$$

so $b_k = 59\times 2^k + 3\times(-4)^k - 11\times 3^k$

check $k=0 \quad 59+3-11 = 51$

$k=1 \quad 118-12-33 = 73$

$k=2 \quad 236+48-99 = \cancel{89}\,185$ ✓

$\underline{W}$ is solution to

$$\left(\begin{array}{ccc|c}4&16&9&185\\2&-4&3&73\\1&1&1&51\end{array}\right)$$

$R_1 \Leftarrow R_1 - 4R_3 \quad R_2 \Leftarrow R_2 - 2R_3$

$$\left(\begin{array}{ccc|c}0&12&5&-19\\0&-6&1&-29\\1&1&1&51\end{array}\right)$$

$R1 \Leftarrow R1 - 5R2 \quad R3 \Leftarrow R3 - R2$

$$\left(\begin{array}{ccc|c}0&42&0&126\\0&-6&1&-29\\1&7&0&80\end{array}\right)$$

$R1 \Leftarrow R1 \times \frac{1}{42}$

$$\left(\begin{array}{ccc|c}0&1&0&3\\0&-6&1&-29\\1&7&0&80\end{array}\right)$$

$R2 \Leftarrow R2 + 6R1 \quad R3 \Leftarrow R3 - 7R1$

$$\left(\begin{array}{ccc|c}0&1&0&3\\0&0&1&-11\\1&0&0&59\end{array}\right)$$

R3 to top

$$\left(\begin{array}{ccc|c}1&0&0&59\\0&1&0&3\\0&0&1&-11\end{array}\right)$$

(c) we want no contribution from $(-4)^k$

so need

$$\begin{pmatrix}t\\13\\2\end{pmatrix} = \begin{pmatrix}4\\2\\1\end{pmatrix}\times k + \begin{pmatrix}9\\3\\1\end{pmatrix}\times l$$

or

$$\left(\begin{array}{cc|c}4&9&t\\2&3&13\\1&1&2\end{array}\right)$$

$$\left(\begin{array}{cc|c}0&5&t-8\\0&1&9\\1&1&2\end{array}\right)$$

$$\left(\begin{array}{cc|c}0&0&t-8-45\\0&1&9\\1&0&-7\end{array}\right)$$

Need $t - 53 = 0$

$\underline{\underline{t = 53}}$

$-7\times 2^k + 9\times 3^k$

$k=0 \quad -7+9 = 2$

$k=1 \quad -14+27 = 13$

$k=2 \quad -28+81 = 53$