

Test 2 2017

Q1 (a)
$$\left(\begin{array}{ccc|ccc} -5 & -1 & 1 & 1 & 0 & 0 \\ 2 & 4 & -3 & 0 & 1 & 0 \\ 3 & -5 & 4 & 0 & 0 & 1 \end{array} \right)$$

$R_2 \leftarrow R_2 + 3R_1$

$R_3 \leftarrow R_3 - 4R_1$

$$\left(\begin{array}{ccc|ccc} -5 & -1 & 1 & 1 & 0 & 0 \\ -13 & 1 & 0 & 3 & 1 & 0 \\ 23 & -1 & 0 & -4 & 0 & 1 \end{array} \right)$$

$R_1 \leftarrow R_1 + R_2$

$R_3 \leftarrow R_3 + R_2$

$$\left(\begin{array}{ccc|ccc} -18 & 0 & 1 & 4 & 1 & 0 \\ -13 & 1 & 0 & 3 & 1 & 0 \\ 10 & 0 & 0 & -1 & 1 & 1 \end{array} \right)$$

$R_3 \leftarrow R_3 / 10$

$$\left(\begin{array}{ccc|ccc} -18 & 0 & 1 & \frac{40}{10} & \frac{10}{10} & 0 \\ -13 & 1 & 0 & \frac{30}{10} & \frac{10}{10} & 0 \\ 1 & 0 & 0 & -\frac{1}{10} & \frac{1}{10} & \frac{1}{10} \end{array} \right)$$

$R_1 \leftarrow R_1 + 18R_3$

$R_2 \leftarrow R_2 + 13R_3$

$$\left(\begin{array}{ccc|ccc} 0 & 0 & 1 & \frac{22}{10} & \frac{28}{10} & \frac{19}{10} \\ 0 & 1 & 0 & \frac{17}{10} & \frac{23}{10} & \frac{13}{10} \\ 1 & 0 & 0 & -\frac{1}{10} & \frac{1}{10} & \frac{1}{10} \end{array} \right)$$

$R_1 \leftrightarrow R_3$

gives $M^{-1} = \frac{1}{10} \begin{pmatrix} -1 & 1 & 1 \\ 17 & 23 & 13 \\ 22 & 28 & 18 \end{pmatrix}$

b) MM^{-1} check

$$= \frac{1}{10} \begin{pmatrix} 5-17+22 & 5-23+28 & 5-18 \\ -2+68-66 & 2-92+84 & 2+52-4 \\ -3-85+88 & 3-115+112 & 3-65+72 \end{pmatrix} = I$$

$$\underline{V} = \frac{1}{10} \begin{pmatrix} -1 & 1 & 1 \\ 17 & 23 & 13 \\ 22 & 28 & 18 \end{pmatrix} \begin{pmatrix} 4 \\ -4 \\ 3 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} -4-4+3 \\ 68-92+39 \\ 88-112+54 \end{pmatrix} = \begin{pmatrix} -5 \\ 15 \\ 30 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 3/2 \\ 3 \end{pmatrix}$$

$$\underline{M}\underline{V} = \begin{pmatrix} -5 & -1 & 1 \\ 2 & 4 & -3 \\ 3 & -5 & 4 \end{pmatrix} \frac{1}{2} \begin{pmatrix} -1 \\ 3 \\ 6 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 5-3+6 \\ -2+12-18 \\ 3-15+24 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ 3 \end{pmatrix}$$

$$\text{Q2 (a) } \det(A) = 1 \times (4 - 5x) - 3 \times (4 - 20) + y \times (2x + 8) \\ = 4 + 5x + 48 + 2xy + 8y$$

$$\text{So } \det(A) = 0 \text{ when } 2xy + 8y + 5x + 52 = 0 \\ (2y + 5)x = -52 - 8y \\ x = \frac{-52 - 8y}{2y + 5}$$

$$\text{unless } y = -\frac{5}{2}, \text{ in which case } \det(A) = 52 + 5x - 5x - 20 \\ = \underline{\underline{32 \neq 0}}$$

$$\text{(or } \det(A) = -3 \times (4 - 20) + 2 \times (2 + 4y) - x \times (-5 - 2y) \\ = 48 + 4 + 8y + 5x + 2xy \text{ as before)}$$

$$\text{(b) } 52 + 5x + 8y + 2xy = 1$$

$$\text{so } x = \frac{-51 - 8y}{2y + 5} \text{ pick } y = -2 \quad x = \frac{-51 + 16}{1} = -35$$

$$\text{(or } y = -3 \quad x = \frac{-51 + 24}{-1} = 27)$$

$$\text{(c) } B = \begin{pmatrix} 1 & 2 & x \\ 3 & 4 & -1 \\ 5 & 6 & y \end{pmatrix}$$

$$\det(B) = x \times (18 - 20) + 1 \times (6 - 10) + y \times (4 - 6)$$

$$= -2x - 4 - 2y$$

Since no xy term
cannot factor and divide.

$$C = \begin{pmatrix} 1 & 2 & x \\ 2 & 4 & 1 \\ 3 & 6 & y \end{pmatrix}$$

$$\det(C) = x \times (12 - 12) - 1 \times (6 - 6) + y \times (4 - 4) = 0$$

(Note $C2 \leftarrow C2 - 2C1 \Rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ rank < 3)