Q1(a)

$$M=\begin{pmatrix}-3 & 9/5\\ \frac{-12}{5} & 6/5\end{pmatrix} \qquad \det(M-\lambda I) = (-3-\lambda)\left(\frac{6}{5}-\lambda\right) - \frac{-12}{5}\times\frac{9}{5} \quad (1)$$

$$= \lambda^2+\lambda\left(\frac{-6}{5}+3\right) + \frac{108}{25} - \frac{18}{5}$$

8/8 (1)

$$= \lambda^2 + \frac{9}{5}\lambda + \frac{18}{25}\lambda$$

$$= \left(\lambda+\frac{6}{5}\right)\left(\lambda+\frac{3}{5}\right) \quad (1)$$

so $\lambda_1 = -\frac{6}{5}$ $\lambda_2 = -\frac{3}{5}$ λ_1 is dominant

$$V_1: \left(\begin{array}{cc|c}-3+\frac{6}{5} & 9/5 & 0\\ -12/5 & 6/5+\frac{6}{5} & 0\end{array}\right) \left(\begin{array}{cc|c}-9/5 & 9/5 & 0\\ -12/5 & \frac{12}{5} & 0\end{array}\right) \begin{array}{l} R_1 \leftarrow R_1\times 5/9 \\ R_2 \leftarrow R_2 + \frac{12}{5}\text{new}(R_1)\end{array} \left(\begin{array}{cc|c}-1 & 1 & 0\\ 0 & 0 & 0\end{array}\right) \quad \underline{v}_1 = \begin{pmatrix}1\\1\end{pmatrix} \quad (1)$$

$$V_2: \left(\begin{array}{cc|c}-3+\frac{3}{5} & 9/5 & 0\\ -12/5 & 6/5+\frac{3}{5} & 0\end{array}\right) \; R_2 \leftarrow R_2 - R_1 \; \left(\begin{array}{cc|c}\frac{-12}{5} & 9/5 & 0\\ 0 & 0 & 0\end{array}\right) \quad \underline{v}_2 = \begin{pmatrix}9\\12\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix}\times 3 \quad (1)$$

$$\text{so } P = \begin{pmatrix}1 & 3\\ 1 & 4\end{pmatrix} \quad P^{-1} = \frac{1}{4-3}\begin{pmatrix}4 & -3\\ -1 & 1\end{pmatrix} = \begin{pmatrix}4 & -3\\ -1 & 1\end{pmatrix} \quad D^k = \begin{pmatrix}(-6/5)^k & 0\\ 0 & (-3/5)^k\end{pmatrix} \quad (1)$$

$$\begin{pmatrix}a_k\\ b_k\end{pmatrix} = \begin{pmatrix}1 & 3\\ 1 & 4\end{pmatrix}\begin{pmatrix}(-6/5)^k & 0\\ 0 & (-3/5)^k\end{pmatrix}\begin{pmatrix}4 & -3\\ -1 & 1\end{pmatrix}\begin{pmatrix}900\\ 1300\end{pmatrix} \quad (1)$$

$$= \begin{pmatrix}(-6/5)^k & 3\times(-3/5)^k\\ (-6/5)^k & 4\times(-3/5)^k\end{pmatrix}\begin{pmatrix}-300\\ 400\end{pmatrix} = \begin{pmatrix}-300\times(-6/5)^k + 1200\times\left(\frac{-3}{5}\right)^k\\ -300\times(-6/5)^k + 1600\times(-3/5)^k\end{pmatrix} \quad (1)$$

(b) $\left(\frac{-6}{5}\right)^k = \left(\frac{-3}{5}\times 2\right)^k = \left(\frac{-3}{5}\right)^k \times 2^k$ so $a_k = 300\times\left(\frac{-3}{5}\right)^k\left(4-2^k\right)$

and $b_k = 100\times\left(\frac{-3}{5}\right)^k\left(16-3\times 2^k\right)$

4/4 so $\left(\frac{-3}{5}\right)^k$ is negative when k is odd and positive when even. (1)

$4-2^k$ is ~~even~~ +ve for $k=0,1$ 0 for $k=2$ −ve for $k\geq 3$ (1)

$16-3\times 2^k$ is +ve for $k=0,1,2$ and −ve for $k\geq 3$

so when $k\geq 3$ a_k and b_k are −ve for k even and +ve for k odd

when $k=0$ both are even, $k=1$ both are odd, when $k=2$ $a_k=0$ and b_k is +ve (2)

Q2)(a) $A = \begin{pmatrix} 3 & 1 \\ 4 & 1 \\ 6 & 1 \\ 7 & 1 \\ 10 & 1 \end{pmatrix}$ $\quad \underline{w} = \begin{pmatrix} -3 \\ 1 \\ 1 \\ 2 \\ 4 \end{pmatrix}$ $\quad A\underline{v} = \underline{w}$

$A^T A \underline{v} = A^T \underline{w}$ $\quad y = mx + b$

1

5/5

so $A^T A = \begin{pmatrix} 3^2+4^2+6^2+7^2+10^2 & 3+4+6+7+10 \\ 3+4+6+7+10 & 1+1+1+1+1 \end{pmatrix} = \begin{pmatrix} 210 & 30 \\ 30 & 5 \end{pmatrix}$ 1

$A^T \underline{w} = \begin{pmatrix} -9+4+6+14+40 \\ -3+1+1+2+4 \end{pmatrix} = \begin{pmatrix} 55 \\ 5 \end{pmatrix}$

1 $\quad$ 1

$$\begin{pmatrix} m \\ b \end{pmatrix} = \frac{1}{1050-900} \begin{pmatrix} 5 & -30 \\ -30 & 210 \end{pmatrix} \begin{pmatrix} 55 \\ 5 \end{pmatrix} = \frac{1}{150} \begin{pmatrix} 275-150 \\ -1650+1050 \end{pmatrix} = \frac{1}{150} \begin{pmatrix} 125 \\ -600 \end{pmatrix} = \begin{pmatrix} 5/6 \\ -4 \end{pmatrix}$$

So $y = \frac{5}{6}x - 4$ 1

(b) $x=3$ $y = \frac{5}{2} - 4 = -\frac{3}{2}$ $\quad x=4$ $y = \frac{10}{3} - 4 = \frac{-2}{3}$ $\quad x=6$ $y = 5-4 = 1$ 1 $\checkmark$ exactly on

(6, 1) on line

$x=7$ $y = \frac{35}{6} - 4 = \frac{11}{6}$ $\quad x=10$ $y = \frac{25}{3} - \frac{12}{3} = \frac{13}{3}$

3/3

Differences: $x=3$ $-3 - \frac{-3}{2} = \frac{-3}{2}$ $\quad x=4$ $1 - \frac{-2}{3} = \frac{5}{3}$ $\quad x=6$ $1-1=0$

$x=7$ $2 - \frac{11}{6} = \frac{1}{6}$ $\quad x=10$ $4 - \frac{13}{3} = \frac{-1}{3}$

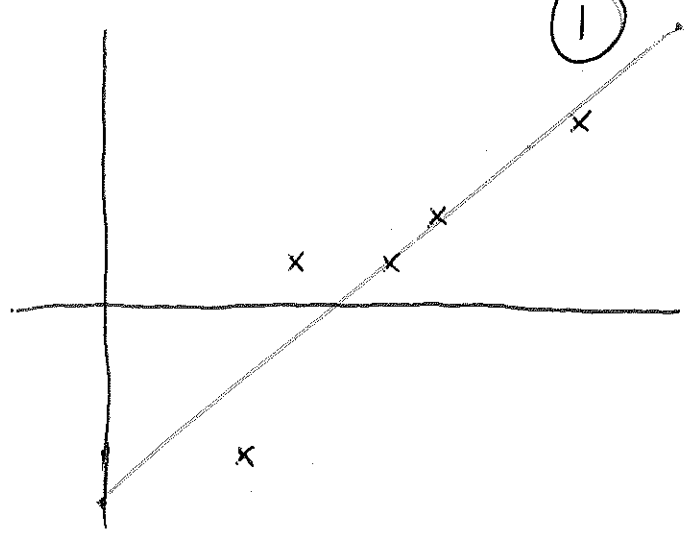
1

$\frac{5}{3} > \frac{3}{2}$, $\frac{10}{6} > \frac{9}{6}$

(4, 1) furthest

Sum differences $= -\frac{3}{2} + \frac{5}{3} + 0 + \frac{1}{6} - \frac{1}{3} = \frac{-9}{6} + \frac{10}{6} + 0 + \frac{1}{6} - \frac{2}{6} = \frac{+1}{6} - \frac{1}{6} = 0$

1



(optional, no marks for graphing)