MATH 1204 test 2

Q1 (a) $\det\begin{pmatrix} 7 & 4 & 2 \\ 12 & 8 & x \\ y-3 & 6 & \end{pmatrix} = 7\times\det\begin{pmatrix} 8 & x \\ -3 & 6 \end{pmatrix} - 4\det\begin{pmatrix} 12 & x \\ y & 6 \end{pmatrix} + 2\times\det\begin{pmatrix} 12 & 8 \\ y & -3 \end{pmatrix}$ (1)

4/4

$= 7\times(48+3x) - 4\times(72-xy) + 2\times(-36-8y)$ (1)

$= 336 + 21x - 288 + 4xy - 72 - 16y$

$= -24 + 21x - 16y + 4xy$ (1)

singular means $4xy - 16y + 21x - 24 = 0$

$(4x-16)y = 24-21x$ (1)

$y = \frac{24-21x}{4x-16}$ unless $x=4$ (1)

when $\det = 60 \neq 0$

(b) $\left(\begin{array}{ccc|ccc} 7 & 4 & 2 & 1 & 0 & 0 \\ 12 & 8 & 3 & 0 & 1 & 0 \\ 13 & -3 & 6 & 0 & 0 & 1 \end{array}\right)$

6/6

$\left(\begin{array}{ccc|ccc} 3 & 3 & 2 & 1 & 0 & 0 \\ 3 & 2 & 4 & 0 & 1 & 0 \\ 1 & 0 & 3 & 0 & 0 & 1 \end{array}\right)$

$R_1 \leftarrow R_1 - 3R_3$ (1)

$R_2 \leftarrow R_2 - 3R_3$

$\left(\begin{array}{ccc|ccc} 0 & 3 & -7 & 1 & 0 & -3 \\ 0 & 2 & -5 & 0 & 1 & -3 \\ 1 & 0 & 3 & 0 & 0 & 1 \end{array}\right)$

$R_1 \leftarrow R_1 - R_2$ (1)

$\left(\begin{array}{ccc|ccc} 0 & 1 & -2 & 1 & -1 & 0 \\ 0 & 2 & -5 & 0 & 1 & -3 \\ 1 & 0 & 3 & 0 & 0 & 1 \end{array}\right)$

$R_2 \leftarrow R_2 - 2R_1$ (1)

$\left(\begin{array}{ccc|ccc} 0 & 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & -1 & -2 & 3 & -3 \\ 1 & 0 & 3 & 0 & 0 & 1 \end{array}\right)$

$R_1 \leftarrow R_1 - 2R_2$ (1)

$R_3 \leftarrow R_3 + 3R_2$

$\left(\begin{array}{ccc|ccc} 0 & 1 & 0 & 5 & -7 & 6 \\ 0 & 0 & -1 & -2 & 3 & -3 \\ 1 & 0 & 0 & -6 & 9 & -8 \end{array}\right)$

$R_2 \leftarrow -R_2 \leftarrow R_3$ (1)

$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -6 & 9 & -8 \\ 0 & 1 & 0 & 5 & -7 & 6 \\ 0 & 0 & 1 & 2 & -3 & 3 \end{array}\right)$ E^{-1}

$\begin{pmatrix} 3 & 3 & 2 \\ 3 & 2 & 4 \\ 1 & 0 & 3 \end{pmatrix}\begin{pmatrix} -6 & 9 & -8 \\ 5 & -7 & 6 \\ 2 & -3 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Final answer (1) (or $\det E^{-1}$ if wrong)

MATH1204 Q2 Test 2

2(a) $XB = 3X - 4C^T$ (1) subtract

$XB - 3X = -4C^T$

6/6 $XB - 3XI = -4C^T$ (1) $\times I$ right identity

$X(B - 3I) = -4C^T$ (1) brackets (right distributivity)

$X = -4C^T(B-3I)^{-1}$ or $4C^T(3I-B)^{-1}$ (1) right inverse

Many ways to do this is 1 mark for all names

$B-3$ is not $B-3I$

If X is $m \times n$ then B must be $n \times p$ to multiply (1)

and to take its inverse B must be $n \times n$.

C^T must be $m \times n$ so C is $n \times m$. (1)

(width XB which is $m \times n$)

(b)

$$3I - B = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 4 \\ -1 & 7 \end{pmatrix} = \begin{pmatrix} -1 & -4 \\ 1 & -4 \end{pmatrix}$$

A/A $4C^T = 4\begin{pmatrix} 3 & 1 \\ 2 & 8 \end{pmatrix}^T = \begin{pmatrix} 12 & 8 \\ 4 & 32 \end{pmatrix}$ (1) transpose

$$(3I-B)^{-1} = \frac{1}{4-4}\begin{pmatrix} -4 & 4 \\ -1 & -1 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} -4 & 4 \\ -1 & -1 \end{pmatrix}$$ (1) inverse

$$\text{So } X = \begin{pmatrix} 12 & 8 \\ 4 & 24 \end{pmatrix}\frac{1}{8}\begin{pmatrix} -4 & 4 \\ -1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & 2 \\ 1 & 8 \end{pmatrix}\begin{pmatrix} -4 & 4 \\ -1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -14 & 10 \\ -12 & -4 \end{pmatrix}$$ (1) matmult

$$= \begin{pmatrix} -7 & 5 \\ -6 & -2 \end{pmatrix}$$

(1) right answer

(-1) for dividing by a matrix