(a) Q1

(1) for writing as matrix

$$\left(\begin{array}{cccc|c} 7 & 3 & 5 & 0 & 11 \\ 6 & -1 & -2 & 5 & -1 \\ -3 & -3 & 1 & -2 & 7 \end{array}\right)$$

$R_1 \leftarrow R_1 - 5R_3$
$R_2 \leftarrow R_2 + 2R_3$ (2)

$$\left(\begin{array}{cccc|c} 22 & 18 & 0 & 10 & -24 \\ 0 & -7 & 0 & 1 & 13 \\ -3 & -3 & 1 & -2 & 7 \end{array}\right)$$

$R_1 \leftarrow R_1 - 10R_2$
$R_3 \leftarrow R_3 + 2R_2$ (2)

$$\left(\begin{array}{cccc|c} 22 & 88 & 0 & 0 & -154 \\ 0 & -7 & 0 & 1 & 13 \\ -3 & -17 & 1 & 0 & 33 \end{array}\right)$$

$R_1 \leftarrow R_1 \times \frac{1}{22}$ (1)

$$\left(\begin{array}{cccc|c} 1 & 4 & 0 & 0 & -7 \\ 0 & -7 & 0 & 1 & 13 \\ -3 & -17 & 1 & 0 & 33 \end{array}\right)$$

$R_3 \leftarrow R_3 + 3R_1$ (1)

$$\left(\begin{array}{cccc|c} 1 & 4 & 0 & 0 & -7 \\ 0 & -7 & 0 & 1 & 13 \\ 0 & -5 & 1 & 0 & 12 \end{array}\right)$$

$x = t$ since it wasn't pivoted

R_1 $w = -7 - 4t$

R_2 $z = 13 + 7t$ (1)

R_3 $y = 12 + 5t$

(b)

$w = -7 - 4t$
$x = 0 + t$
$y = 12 + 5t$
$z = 13 + 7t$

$7w + 3x + 5y$
$= -49 - 28t + 3t + 60 + 25t$
$= 11$ ✓

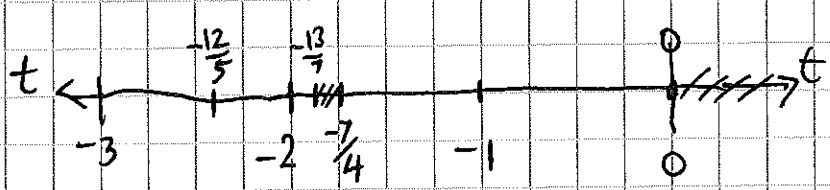
$6w - x - 2y + 5z$
$= -42 - 24t - t - 24 - 10t + 65 + 35t$
$= -1$ ✓ (1)

$-3w - 3x + y - 2z$
$= 21 + 12t - 3t + 12 + 5t - 26 - 14t$
$= 7$ ✓

$x \geq 0$ when $t \geq 0$
$w \geq 0$ when $t \leq -\frac{7}{4}$
only one of these since $-\frac{7}{4} < 0$

$y \geq 0$ when $t \geq -\frac{12}{5}$

$z \geq 0$ when $t \geq -\frac{13}{7}$ (1)



so when t is between $-\frac{13}{7}$ and $-\frac{7}{4}$ 3 are positive, but not x.

eg $t = -\frac{9}{5} = -1.8$

Q2)

$$\left(\begin{array}{ccc:c} 6 & 6 & 7 & 0 \\ 7 & 2 & 3 & 6 \\ 6 & 5 & 6 & 1 \end{array}\right)$$

$R_1 \Leftarrow R_1 - R_3$ create a 1 and 0 (1)

$$\left(\begin{array}{ccc:c} 0 & 1 & 1 & -1 \\ 7 & 2 & 3 & 6 \\ 6 & 5 & 6 & 1 \end{array}\right)$$

Pivot $R_2 \Leftarrow R_2 - 2R_1$ (2)

$R_3 \Leftarrow R_3 - 5R_1$

$$\left(\begin{array}{ccc:c} 0 & 1 & 1 & -1 \\ 7 & 0 & 1 & 8 \\ 6 & 0 & 1 & 6 \end{array}\right)$$

Pivot

$R_2 \Leftarrow R_2 - R_3$ (2)

$R_1 \Leftarrow R_1 - R_3$

$$\left(\begin{array}{ccc:c} -6 & 1 & 0 & -7 \\ 1 & 0 & 0 & 2 \\ 6 & 0 & 1 & 6 \end{array}\right)$$

Pivot

$R_1 \Leftarrow R_1 + 6R_2$ (2)

$R_3 \Leftarrow R_3 - 6R_2$

$$\left(\begin{array}{ccc:c} 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -6 \end{array}\right)$$

(1)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ -6 \end{pmatrix} \qquad \begin{pmatrix} 6 & 6 & 7 \\ 7 & 2 & 3 \\ 6 & 5 & 6 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \\ -6 \end{pmatrix} = \begin{pmatrix} 12+30-42 \\ 14+10-18 \\ 12+25-36 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$$ as required

Q3)

(For $BC=CB$ B must be 2×2 to multiply left and right)

$$C=\begin{pmatrix}3 & 1\\ -2 & 0\end{pmatrix}$$

Hard way

(1) for a correct answer

(1) for showing how it was found or deduced (not guessed)

$$B=\begin{pmatrix}p & q\\ r & s\end{pmatrix} \qquad CB=\begin{pmatrix}3p+r & 3q+s\\ -2p & -2q\end{pmatrix}$$

$$BC=\begin{pmatrix}3p-2q & p\\ 3r-2s & r\end{pmatrix}$$

so $r=-2q \quad p=3q+s \quad -2p=3r-2s$

(so $-2p=-2(3q+s)=-6q-2s=3r-2s$)

so if we pick $q=3$ $r=\cancel{2}-6$ then $s=p-3q=p-9$

and $p=3q+s=9+s$

so if we take $p=10$ $s=1$ it should work

$$\begin{pmatrix}10 & 3\\ -6 & 1\end{pmatrix}\begin{pmatrix}3 & 1\\ -2 & 0\end{pmatrix}=\begin{pmatrix}24 & 10\\ -20 & -6\end{pmatrix}$$

$$\begin{pmatrix}3 & 1\\ -2 & 0\end{pmatrix}\begin{pmatrix}10 & 3\\ -6 & 1\end{pmatrix}=\begin{pmatrix}24 & 10\\ -20 & -6\end{pmatrix}$$ ✓

or Easy way

$$B=C^2=\begin{pmatrix}3 & 1\\ -2 & 0\end{pmatrix}\begin{pmatrix}3 & 1\\ -2 & 0\end{pmatrix}=\begin{pmatrix}7 & 3\\ -6 & -2\end{pmatrix} \qquad BC=C^3=CB$$

check $$BC=\begin{pmatrix}3 & 1\\ -2 & 0\end{pmatrix}\begin{pmatrix}7 & 3\\ -6 & -2\end{pmatrix}=\begin{pmatrix}15 & 7\\ -14 & -6\end{pmatrix} \qquad CB=\begin{pmatrix}15 & 7\\ -14 & -6\end{pmatrix}$$ ✓