

Math1204 Test 3

March 6th 2013

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The questions are weighted as shown and can be answered in any order.

Suppose we have the following relations between two series and are given that $a_0 := 80$ and $b_0 := 24$:

$$\begin{aligned}a_{n+1} &:= \frac{33}{4} a_n - \frac{35}{2} b_n \\ b_{n+1} &:= 3 a_n - \frac{25}{4} b_n\end{aligned}$$

1. Determine the values of a_1 , a_2 and a_3 using the relations directly. [3]
2. Use diagonalisation to find the general formula for a_n and b_n . Check your answers for $n = 2$ and $n = 3$. [11]
3. Explain why both sequences are increasing as n increases. If a_0 was instead 70, what value would b_0 have to be for a_n to tend towards 0 as n increases? Why? [3]
4. Use algebra to investigate the eigenvalues of a general 2×2 matrix with rank 1 and hence or otherwise give a matrix E (without any 0s or 1s) which has $E^n = E$ for every positive integer n . [3]