Cape Breton University

Math 1204

MATRIX ALGEBRA

February 2013

Time : $\frac{3}{2}$ hours

Please answer any THREE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. (a) Evaluate $\det(E - \lambda I)$ using a cofactor expansion to get all three eigenvalues. [6]

$$E := \begin{pmatrix} -22 & -16 & 4 \\ 39 & 27 & -6 \\ 0 & -4 & 4 \end{pmatrix}$$

(b) Find an eigenvector for E which contains no fractions using eigenvalue $\lambda = 2$. [3]

(c) Calculate one of the other eigenvectors of E. [2]

Q2. (a) Write these equations in matrix form and use row operations to get them to an equivalent of reduced row echelon form. What is the rank of the underlying matrix? [6]

$$\begin{aligned} v + w + z &= 4 \\ v + 2\,x + 2\,y + z &= 7 \\ v + w - y + z &= 2 \\ w - 2\,x + 2\,y &= 5 \\ v + 2\,x + y + z &= 5 \end{aligned}$$

(b) Find two homogeneous solutions which are not multiples of each other and a combination of these two solutions which contains 3 zeros. Check all of them against the original equations. Why is it impossible to have a particular solution with more than two zeros in? [5]

Q3. (a) Find the adjoint of F. [5]

$$F := \begin{pmatrix} 4 & x & -3 \\ y & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(b) Multiply your answer by F to check if you are correct, deduce $\det(F)$ and give F^{-1}. [4]

(c) Why is there no value for x or y such that F is guaranteed non-singular? Find two different pairs of values for x and y which would give $\det(F) = 7$. [2]

Q4. (a) What are the eigenvalues and eigenvectors of the matrix $M := \begin{pmatrix} -3 & 4 \\ -1 & -8 \end{pmatrix}$? [3]

(b) Evaluate $M^2 + 28I$ and relate it to M. [2]

(c) Define $N := \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$. Evaluate and factor $N^2 + \det(N) \times I$ in terms of N. [3]

(d) Under what circumstances will N not have two eigenvectors? [3]

END OF QUESTION PAPER