Cape Breton University

Math 1204

MATRIX ALGEBRA

April 2013 | Time : 3 hours

Please answer any FIVE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. We have the following six data points; (-5,1), (-1,2), (0,1), (1,-1), (2,-2), (3,3)

(a) Find the best fit quadratic equation through this data. [8]

(b) Re-use your working to identify the equation of the best fit straight line. [2]

(c) Sketch the points and both best fit equations on the same graph and predict which data points are the furthest from each. Find the exact distance of your predicted points from their fitted values. [2]

Q2. (a) Given that one eigenvector of the matrix $B := \begin{pmatrix} -18 & -102 & 246 \\ 15 & 75 & -174 \\ 5 & 24 & -55 \end{pmatrix}$ is $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$, find one eigenvalue and then the other two eigenvalues. [5]

(b) Find an eigenvector of B that does not belong to a dominant eigenvalue by using row operations. [5]

(c) Explain several sorts of different behaviour we could expect for $B^k v_0$ as k increases with respect to these particular eigenvalues and different choices for v_0. [2]

Q3. (a) Which vector space axioms are false or true for these two different sets of points in the (x, y) plane? (give reasons or counterexamples for each axiom/set) [5]

$$(i)\ \ y \leq 2x + 1 \qquad\qquad (ii)\ \ y > |x|$$

(b) Create two lines which are wholly within in the plane $P := \begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = 11$ but which do not ever intersect. [2]

(c) Find the equation of a line L within P which is perpendicular to your lines from (b) and determine where L intersects with them both, and hence determine the distance between the lines. [5]

Q4. (a) Evaluate the determinant of $M := \begin{pmatrix} 4 & 2 & 3 \\ 3 & -7 & x \\ y & 2 & 3 \end{pmatrix}$ and explain why there is no value of x or y which guarantees that M is non-singular. [5]

(b) Calculate the the inverse of M and multiply your answer by M to check. [7]

Q5. In this question we will be studying the recurrence $c_{j+1} := 7c_j - 36c_{j-2}$ where $c_0 = 41$, $c_1 = -211$ and $c_2 = 203$.

(a) Substitute the given values to get c_3 and c_4. [2]

(b) Find the three different eigenvalues underlying this recurrence and then use the diagonalisation method to get the formula for c_n for any n. [8]

(c) Use logarithms to predict for which value of j we will first have $c_j > 70000000$ and check whether it is accurate. [2]

Q6. (a) Simplify this equation $E(XA + 3I) + 2A^T A = 0$ to find X in simplest terms, assuming inverses exist where needed and quoting which algebra rules you use. [4]

(b) Find the inverses of both of these matrices and substitute them into your solution for (a) to see if you get maximum absolute value 13 for the elements of X. [3]

$$A := \begin{pmatrix} 4 & 5 \\ -1 & -2 \end{pmatrix} \quad , \quad E := \begin{pmatrix} 5 & 6 \\ 3 & 4 \end{pmatrix}$$

(c) If A was a 1×2 matrix what sizes must X and E be for the original equation in (a) to possibly hold? Find matrices A and X of these sizes which imply that no E satisfying the equation can exist, explaining why. [5]

Q7. This question will involve these vectors:

$$v_1 := \begin{pmatrix} 1 \\ -1 \\ -2 \\ 1 \end{pmatrix} \quad , \quad v_2 := \begin{pmatrix} -4 \\ 3 \\ 1 \\ 4 \end{pmatrix} \quad , \quad v_3 := \begin{pmatrix} -2 \\ -1 \\ -3 \\ 2 \end{pmatrix} \quad , \quad v_4 := \begin{pmatrix} 3 \\ 2 \\ 1 \\ 3 \end{pmatrix}$$

(a) Verify that the distances from v_2 to v_4 and v_3 to v_4 are the same and determine whether v_1 is closer to v_3 or v_4. Why is the sum of the distances from v_3 to v_1 and v_1 to v_4 nearly the same as the distance directly between v_3 and v_4? [3]

(b) Show that the four vectors are not independent in $\mathbb{R}^4$. [4]

(c) Find the unique vector which is perpendicular to the four vectors and hence find the dot product equation of the hyperplane they span. [5]

END OF QUESTION PAPER