

# Math1204 Test 4

March 14<sup>th</sup>, 2012

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The parts of the questions are weighted as shown and can be answered in any order.

- Evaluate  $c_3$  if  $c_0 := 1$ ,  $c_1 := 74$  and  $c_2 := 48$  and  $c_{n+1} := 3c_n + \frac{7}{4}c_{n-1} - \frac{15}{2}c_{n-2}$ . [1]
  - One eigenvalue of the matrix underlying the recurrence is 2; find the other eigenvalues and form the matrices  $D^k$  and  $P$  using them. [4]
  - Solve the system of equations

$$PX = \begin{pmatrix} 48 \\ 74 \\ 1 \end{pmatrix}$$

for  $X$  using row operations and explain why you can then write

$$\begin{pmatrix} c_{k+2} \\ c_{k+1} \\ c_k \end{pmatrix} = PD^k X$$

Evaluate this expression to get a formula for  $c_k$ . [4]

- Why, despite the fractions in the eigenvalues, are  $c_0$  to  $c_4$  integers? [1]
  - Why is it approximately true that  $c_{k+5}$  is nearly 100 times  $c_k$  for large  $k$ ? Which value of  $k$  is the *last* for which  $c_{k+5} < 70c_k$ ? [2]
- What is the best fit straight line for this data? [5]

x	-4	-2	-2	-1	7	8
y	9	-5	9	0	-3	-5

- Determine all vertical distances from your best fit line to the data points and verify that the sum of these differences is zero. Which data point is closest to your line? [3]