Math1204 Test 3

February 15^{th}, 2012

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The parts of the questions are weighted as shown and can be answered in any order.

1. (a) By using a well chosen column operation on the appropriate matrix, find all three eigenvalues of matrix M. [5]

$$M := \begin{pmatrix} -7 & 10 & -3 \\ 24 & -26 & 12 \\ 78 & -90 & 38 \end{pmatrix}$$

   (b) Verify by multiplying that $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is an eigenvector of M and find its eigenvalue, then find one of the two other eigenvectors of M. [5]

2. (a) Use the adjoint method to find the inverse of this matrix: [6]

$$Q := \begin{pmatrix} 2 & 1 & -1 \\ 0 & -6 & 2 \\ 5 & 1 & 4 \end{pmatrix}$$

   (b) Create a 2×2 matrix P containing only positive numbers which has determinant equal to -1. What is P^{-1}? How many negative numbers are in P^2 and P^{-2}? How many negative numbers will be in P^k for any given integer k? [4]