

# Math115 Test 5: Geometrical Vectors

March 23rd, 2010

Answer all parts of the questions and give complete reasons and checks for your answers. Attach all rough working and do not erase anything that might earn you marks. The parts of the questions are weighted as shown in square brackets on the right.

1. (a) Find the vector orthogonal to both direction vectors of these lines and hence find the shortest distance between them: [7]

$$L_1 := \begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix} + k \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad L_2 := \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix} + j \begin{pmatrix} 4 \\ -1 \\ 6 \end{pmatrix}$$

- (b) Give a dot product equation for a plane  $P_1$  which never meets  $L_1$ . Give another plane  $P_2$ , also in dot product form, which contains  $L_2$  completely. Verify your answers using algebra. Could  $P_1$  and  $P_2$  be the same plane? [3]
2. (a) These are hyperplanes in 4 dimensional space; find an equation of all the points at which they intersect and explain what kind of object their intersection is. [5]

$$H_1 : \begin{pmatrix} -5 \\ 1 \\ 0 \\ 2 \end{pmatrix} \circ \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = 4, \quad H_2 : \begin{pmatrix} 1 \\ -4 \\ -3 \\ 5 \end{pmatrix} \circ \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = -7$$

- (b) Find the points of intersection of this line  $L$  with each of  $H_1$  and  $H_2$ . Which point of intersection is nearer to the origin? [5]

$$L := \begin{pmatrix} 1 \\ -2 \\ -3 \\ -1 \end{pmatrix} + t \begin{pmatrix} 4 \\ 3 \\ -1 \\ 2 \end{pmatrix}$$