Cape Breton University

MATH115

Matrix Algebra

April 2009

Time : 3 hours

Please answer any FIVE of these questions, making sure to give all checks, reasoning and working for all questions answered.

Q1. This matrix A is used to describe the population of a group of deer; q_n is the number of adults after n years and j_n the number of juveniles:

$$A := \begin{pmatrix} \frac{1}{5} & \frac{1}{2} \\ 2 & \frac{1}{5} \end{pmatrix} \quad , \quad \begin{pmatrix} j_n \\ q_n \end{pmatrix} = A \begin{pmatrix} j_{n-1} \\ q_{n-1} \end{pmatrix} \quad , \quad q_0 := 150 \ , \quad j_0 := 200$$

(a) Determine the number of adults and juveniles after 1 year. [1]

(b) Use diagonalisation to find the formula for the populations after n years. [9]

(c) What ratio does the number of adults to juveniles tend to as n increases? Will the population go extinct? [2]

Q2. (a) Find all eigenvectors of matrix B: [6]

$$B := \begin{pmatrix} 5 & 0 & 4 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

(b) Create a related 4×4 matrix with the same eigenvalues, with both having multiplicity 2, explaining why this will be the case. [2]

(c) Use Gram-Schmidt to get a pair of orthogonal eigenvectors belonging to the eigenvalue of multiplicity 2 which have no zeroes in. [4]

Q3. (a) Re-arrange these equations into a standard order, form them into a matrix and then get them to row-echelon form to find all solutions. [7]

$$4y + z = 2x - 3(w+1) \quad , \quad 4w + 3(y+1) = 2x - z \ ,$$
$$2(w+x) = 4y + z - 2 \quad , \quad 3y + z = 2(x - 3w) - 5$$

(b) Find the best fit straight line through these points and draw a graph containing the points and the line: (3,1), (1,-1), (2,3), (-1,-2). [5]

Q4. We are given this matrix:

$$E := \begin{pmatrix} -1 & -1 & -2 \\ -5 & 2 & 5 \\ 3 & 2 & 4 \end{pmatrix}$$

(a) Check that the determinant of E is -1. [2]

(b) Find E^{-1} using the adjoint method. [5]

(c) Check that $\det(E^{-1})$, $\det(E^2)$ and $E^T(E^{-1})^T$ have the expected values and explain why these relations hold, algebraically. [5]

Q5. Line L is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + k \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and plane P has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} = 9$

(a) Where does line L intersect plane P? [3]

(b) How close does L get to P's normal vector that passes through the origin? [9]

Q6. (a) What is the determinant of $H := \begin{pmatrix} 2 & x & 1 \\ 1 & -1 & 2 \\ -1 & -2 & y \end{pmatrix}$? [3]

(b) What value of x guarantees that H is non-singular? [3]

(c) If $y = x$ when is H singular? [2]

(d) What rank does H have if $y = 7$ and $x = -1$? [2]

(e) Could the rank of H ever be 1? [2]

Q7. (a) If $G := \begin{pmatrix} 6 & -18 \\ 2 & -7 \end{pmatrix}$ check that G and G^T have the same eigenvalues but different eigenvectors. Explain why any matrix and its transpose will share eigenvalues. [7]

(b) Solve this equation for X, explaining assumptions, and factorise your answer: [5]

$$(AX - 5CB^T) = (BA^T)^T$$

END OF QUESTION PAPER