Math115 Test 4: Recurrence and Curve Fitting (again)

Answer each question on a new sheet of paper, and do not erase anything. Show all working, reasoning and checks to achieve full marks. The number in square brackets indicates the number of marks available for each part of each question. Should you require a hint one may be given in return for a mark.

1. Two number sequences start with $c_1 := 5184$ and $d_1 := 6912$ and then are inter-related by these two equations:

$$24c_{n+1} - 34d_n = c_n , \quad 48d_{n+1} - 17c_n = 2d_n$$

   (a) Find c_2 and d_2. [1]

   (b) Set up the matrix equation for this system identifying what power of the matrix will give you c_i and d_i when multiplied by the initial values. [2]

   (c) Diagonalise the matrix and hence find the formulae for c_i and d_i. [8]

   (d) Can c_i ever be lower than d_i when $i > 1$? Will d_i ever be negative? What value does c_i tend towards as i goes to infinity? [2]

2. (a) Find the quadratic curve which best fits through these points. [5]

x_i	2	1	0	-1	-2
y_i	3	1	-1	2	5

   (b) Plot the points and the curve you have just found. How close does the curve come to a data point? [2]