

### Math115 Test 3: Recurrence and Curve Fitting

Answer each question on a new sheet of paper, and do not erase anything. Show all working, reasoning and checks to achieve full marks. The number in square brackets indicates the number of marks available for each part of each question. Should you require a hint one may be given in return for a mark.

1. A sequence of numbers starts with  $a_0 := 114$ ,  $a_1 := 79$  and  $a_2 := 177$  and all remaining numbers are formed using  $a_{n+1} := 4a_n + 7a_{n-1} - 10a_{n-2}$ .
  - (a) Form the underlying matrix of the relation and check that 1 is an eigenvalue of it and its eigenvector has the special recurrence form. [2]
  - (b) Find the other two eigenvectors and make  $P$ , the eigenvector matrix. [2]
  - (c) Get a relation between a vector including  $a_n$ , the diagonalisation of the underlying matrix and a vector  $v_0$  including  $a_1$  and  $a_0$ . [2]
  - (d) Find which vector is the solution  $w$  for  $Pw = v_0$  and hence or otherwise find the formula for  $a_n$  in general. [6]  
[you should not have to find the inverse of  $P$  this way, despite diagonalisation]
  - (e) Check your predicted value for  $a_3$  and explain why all values of  $a_n$  after this will be larger than their predecessor from this point on. [2]
2. Find the quadratic curve which fits through these points  $(x_i, y_i)$  exactly; [6]

$$(2, 7), (-1, 4) \text{ and } (-3, -3).$$