Cape Breton University

Math115

Matrix Algebra

April 2008

Time : 3 hours

Answer FIVE of the SEVEN questions, giving all working and reasoning.

Q1. (a) Find all solutions of this system of equations using Reduced Row Echelon Form. [8]

$$\left(\begin{array}{cccc|c} 3 & -1 & -5 & -4 & 8 \\ -1 & 2 & 4 & 1 & -5 \\ 0 & -5 & -7 & 1 & 7 \\ -2 & -6 & 6 & -2 & -8 \end{array}\right)$$

(b) Identify a solution point which has exactly one zero in it, and one which has two zeroes. Explain why no solution can have more than two in this case. How many zeros can be in a solution for a general 4×4 matrix system? [4]

Q2. (a) Find both a best fit straight line and a best fit quadratic through the points (0,-2), (1,10), (-1,-7) and (-2,5). [9]

(b) Show that none of the four points are closer to the straight line than the curve. Could this ever not be the case? [3]

Q3. (a) Given $(2C + AX)^T = 3B$, use the rules of matrix algebra to solve it in terms of X, stating any rules or assumptions used at each step. [4]

(b) Given these matrices, find X according to your solution to (a) using the row operation method to find A^{-1}. [5]

$$B := C := \begin{pmatrix} 2 & 4 & 5 \\ 4 & -2 & -1 \\ 5 & -1 & 2 \end{pmatrix}, \quad A := \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

(c) Explain three essentially different conditions on the matrices A, B or C such that X could have no solution at all. [3]

Q4. (a) Verify that $\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ is an eigenvector of $E := \begin{pmatrix} -18 & -29 & -75 \\ 3 & -10 & -15 \\ 3 & 11 & 24 \end{pmatrix}$ and deduce its eigenvalue. [2]

(b) Expand $\det(E - \lambda I)$ and divide through by the known eigenvalue to find the other two eigenvalues. [6]

(c) Find one of the other two eigenvectors. [4]

Q5. (a) These two sequences both have the same dominant eigenvalue. Use this and the special eigenvector formula to efficiently diagonalise and find a_n and b_n. [10]

$$a_{i+1} = 9a_i - 14a_{i-1}\ , \quad a_0 = 3\ , \quad a_1 = 1\ , \quad b_{i+1} = 4b_i + 21b_{i-1}\ , \quad b_0 = 2\ , \quad b_1 = -16$$

(b) Which of the two sequences will have the larger absolute value when n is odd? What will the ratio of $\frac{a_n}{b_n}$ tend to as n tends to infinity? [2]

Q6. (a) Find where these two planes intersect. Check your answer lies on both planes. [4]

$$P_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} = 10\ , \quad P_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 2$$

(b) Where does this line intersect each plane? [5]

$$L : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} + k \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

(c) What is the distance between the two points of intersection in (b)? [3]

Q7. (a) Find the inverse of this matrix $H := \begin{pmatrix} 4 & 3 & -2 \\ -5 & x & 3 \\ 2 & 1 & -1 \end{pmatrix}$ using the adjoint method. [6]

(b) Explain why H is never singular for any value of x. [2]

(c) Create a 3×3 matrix which is never singular despite having both an x and a y in and explain why such a 2×2 matrix is impossible. [4]

END OF QUESTION PAPER