Math115 Test3: Diagonalisation

1. (a) Given $A := \begin{pmatrix} 50 & 36 \\ -72 & -52 \end{pmatrix}$, calculate A^2 directly, and then diagonalise A to get a general formula for A^k in terms of powers of its eigenvalues. [8]

   (b) Check your answer with $k = 0$ and $k = 2$. [2]

2. The matrix B is defined as

$$\begin{pmatrix} -2 & 2 & -2 \\ 6 & 4 & 3 \\ -14 & -18 & -5 \end{pmatrix}$$

   (a) Find the eigenvector of B corresponding to the eigenvalue -3. [6]

   (b) Find the eigenvalue of B corresponding to the eigenvector $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$. [2]

   (c) Using algebra, and not repeating previous work, explain why kB will share all eigenvectors with B but the eigenvalues will be k times the eigenvalues of B. [2]