Math115 Test1: Row Operations and Matrix Algebra

January 26, 2006

1. (a) Perform row operations to solve this system of equations by taking the associated matrix to reduced row echelon form.

$$\begin{aligned} 2w + y - x - z &= 11 \\ 3y - 3z - w - 3x &= 5 \\ 2w + 3x - 3y - 2z &= 9 \end{aligned}$$

   (b) From your answer deduce and verify the general solution to the homogeneous equation

$$\begin{pmatrix} 2 & -1 & 1 & -1 \\ -1 & -3 & 3 & -3 \\ 2 & 3 & -3 & -2 \end{pmatrix} V = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

2. Given these matrices, determine these compound matrices or explain why they do not exist.

$$P := \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 2 & 1 \end{pmatrix} \quad , \quad Q := \begin{pmatrix} 4 & 0 \\ 3 & -1 \end{pmatrix}$$

   (a) i. $2P + 3Q$
   ii. PQ^T
   iii. $(Q + P^T P)^{-1}$
   iv. PP^T
   v. P^2
   vi. Q^3

   (b) i. What size would a matrix R have to be for QRP to be a matrix?
   ii. Give such an R for which QRP has rank less than 2.