Cape Breton University

Math115

MATRIX ALGEBRA

April 2006

Time : 3 hours

Answer FIVE of the SEVEN questions, giving all working and reasoning.

Q1. (a) Find the eigenvalue of multiplicity 2 in this matrix $A := \begin{pmatrix} 103 & -48 & -48 \\ 120 & -53 & -60 \\ 96 & -48 & -41 \end{pmatrix}$. [5]

(b) Find two eigenvectors belonging to this eigenvalue and check they are independent and satisfy the eigenvector-eigenvalue equation. [5]

(c) If, in general, two eigenvectors v_1 and v_2 share an eigenvalue determine which real numbers c_i make $v := c_1v_1 + c_2v_2$ an eigenvector of this eigenvalue too. [2]

Q2. (a) Find the inverse of this matrix using carefully chosen row operations: [9]

$$E := \begin{pmatrix} 2 & -2 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ -2 & -2 & -1 & -1 \\ 1 & -1 & 0 & 1 \end{pmatrix}$$

(b) Multiply E and your answer to see how close you got to the inverse. [1]

(c) In general, if $A^{-1}(XC^T)^T = B$ what is X in terms of A, B and C? [2]

Q3. The values in two sequences are related by the following two equations:

$$p_{i+1} := \frac{1}{2}p_i + 5q_i \quad , \quad q_{i+1} := \frac{5}{3}p_i - \frac{1}{3}q_i$$

(a) Diagonalise the underlying matrix and hence get an expression for p_n and q_n in terms of p_0 and q_0. [10]

(b) Show that if $p_0 := 26$ and $q_0 := 13$ then p_n and q_n always stay in the ratio 2:1 and deduce another simple ratio with the same property. [2]

Q4. (a) What is the best fit line to this data? [5]

x_i	3	2	1	0	-1
y_i	-2	1	3	3	-7

(b) What is the best fit quadratic to the data? [5]

(c) At which points in the xy plane do your line and quadratic meet? [2]

Q5. (a) Find the line in $\mathbb{R}^4$ which is a solution to this system of equations: [9]

$$\begin{aligned} 9w + 6x &= 15y + z - 4 \\ 2w + 4x &= 10y - 2z \\ 4w + 6x + 1 &= 12y - z \\ 5w + 3 &= 3y + 2z \end{aligned}$$

(b) Check whether the point with $w = 4$, $x = 2$, $y = 3$ and $z = 7$ is on your line. [1]

(c) What space is perpendicular to your line and passes through the point $\begin{pmatrix} 4 \\ 3 \\ -5 \\ -2 \end{pmatrix}$? [2]

Q6. (a) Evaluate the determinant of this matrix: [5]

$$F := \begin{pmatrix} -2 & 3 & 5 \\ y & -6 & -8 \\ -9 & x & 4 \end{pmatrix}$$

(b) What is the value of y for any given x which makes the determinant zero? [2]

(c) Taking the columns of F now as vectors, and letting $x = -2$, find the value of y which makes the first two columns orthogonal. [1]

(d) Use the two orthogonal vectors with Gram Schmidt to find an orthogonal basis. [4]

Q7. (a) Find a basis for the space spanned by these vectors using vanishing equations.

$$\begin{pmatrix} 7 \\ 3 \\ -4 \end{pmatrix}, \quad \begin{pmatrix} -4 \\ -2 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 8 \\ 2 \\ -1 \end{pmatrix}, \quad \begin{pmatrix} 7 \\ 1 \\ 1 \end{pmatrix}$$

[5]

(b) Give a set of three different vectors from $\mathbb{R}^3$ which have a basis containing just one vector and explain why all such sets will be similar. [2]

(c) Check all three vector subspace axioms for this set: $S := \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : \frac{x}{y} \neq 1 \right\}$. [5]

END OF QUESTION PAPER