Math 115 Test 5: Sequences, Planes and Subspaces

March 23, 2005

Each question is weighted as shown in square brackets, use the appropriate amount of time and space to answer all parts. Give *all* working and reasoning for two of the three questions to achieve full marks.

1. (a) A recurrence is given as $a_{i+1} + 20a_{i-2} := 2a_i + 19a_{i-1}$. Write it as a 3×3 matrix relating $\begin{pmatrix} a_{i+1} \\ a_i \\ a_{i-1} \end{pmatrix}$ to $\begin{pmatrix} a_i \\ a_{i-1} \\ a_{i-2} \end{pmatrix}$ by multiplication. [2]

   (b) Find the eigenvalues of the matrix and determine which one is dominant. [6]

   (c) Use what we discussed in the lecture to deduce the eigenvectors and verify that they satisfy the eigenvector equation as expected. [2]

2. (a) i. What is the equation of the line that passes through the point $q := \begin{pmatrix} -4 \\ 3 \\ -5 \end{pmatrix}$ and is normal to the plane P with equation $-3x + 2y - 6z = -1$? [2]

   ii. Where does the line intersect with P? [2]

   iii. What is the distance between q and P along this line? [2]

   (b) Verify that $e_1 := \begin{pmatrix} 2 \\ 1 \\ 3 \\ -2 \end{pmatrix}$ and $e_2 := \begin{pmatrix} -1 \\ 1 \\ 3 \\ 4 \end{pmatrix}$ are orthogonal and use $v_3 := \begin{pmatrix} 2 \\ 1 \\ 4 \\ 4 \end{pmatrix}$ to find a vector orthogonal to both e_1 and e_2. [4]

3. (a) Check each of the three vector subspace properties for these three sets and determine which properties are true for which sets, either by showing vectors which void the property or by proving that all possible combinations work. [9]

$$\left\{ \begin{pmatrix} x \\ y \end{pmatrix} ; xy \geq 1 \right\} , \quad \left\{ \begin{pmatrix} x \\ y \end{pmatrix} ; x < 2 \text{ or } y < 1 \right\} , \quad \left\{ \begin{pmatrix} x \\ y \end{pmatrix} ; y \leq 2x - 5 \right\}$$

   (b) Which 2 of these sets when combined make an actual vector space? [1]