Math 115 Test 2: Inverses and Properties

February 2, 2005

Each question is weighted as shown in square brackets, use the appropriate amount of time and effort to answer all parts. Give all working and reasoning for your answers to achieve full marks.

1. (a) Find the inverse of M by augmenting the identity and finding the reduced row echelon form. [7]

$$M := \begin{pmatrix} -1 & 4 & -4 \\ 1 & -3 & 1 \\ 1 & -2 & 0 \end{pmatrix}$$

   (b) Deduce the solution to the homogeneous equation $MX = 0$, giving your reasons. [1]

   (c) Use determinant operations and a Laplace expansion to calculate the determinant of M. [2]

2. We define a matrix X as self-inverse if it satisfies $X^2 = I$.

   (a) Check that W is self-inverse but find a second self-inverse matrix whose product with W is not a self-inverse matrix. [4]

$$W := \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}$$

   (b) Use matrix algebra rules to prove that if A is self-inverse then A^T is too. [3]

   (c) Explain why all self-inverse matrices must be square, why the identity is always self-inverse and find a 3x3 self-inverse matrix which is not the identity matrix. [3]