Math115 Test1: Row Operations and Matrix Algebra

January 19, 2005

1. (a) Perform row operations to solve the matrix equation. [11]

$$MX = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 3 & 1 \\ 1 & -1 & -2 & 2 \\ 0 & 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 6 \\ -1 \\ 6 \end{pmatrix} = Y$$

   (b) Without doing any more row operations, explain the value of the rank of M. [2]

   (c) What is the rank of Y? Can any matrix with the same dimensions have smaller rank? [2]

2. (a) Solve this matrix algebra expression for W giving all steps and reasoning; [7]

$$(s(A + WB))^{-1} = C^T C$$

   (b) Substitute these values into your answer to get W, verifying that all matrix inverses calculated satisfy the relation $ZZ^{-1} = I$. [8]

$$s := \frac{1}{15}, \quad A := \begin{pmatrix} 2 & -22 \\ -44 & 3 \end{pmatrix}, \quad B := \begin{pmatrix} 5 & 2 \\ -2 & -1 \end{pmatrix}, \quad C := \begin{pmatrix} -2 & -2 \\ -1 & -1 \\ 0 & 1 \end{pmatrix}$$